

The final stage of gravitationally collapsed thick matter layers

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In the presence of a *minimal length* physical objects cannot collapse to an infinite density, singular, matter point. In this note we consider the possible final stage of the gravitational collapse of “thick” matter layers. The energy momentum tensor we choose to model these shell-like objects is a proper modification of the source for “non-commutative geometry inspired”, regular black holes. By using higher momenta of Gaussian distribution to localize matter at finite distance from the origin, we obtain new solutions of the Einstein’s equation which smoothly interpolates between Minkowski geometry near the center of the shell and Schwarzschild spacetime far away from the matter layer. The metric is curvature singularity free. Black hole type solutions exist only for “heavy” shells, i.e. $M \geq M_e$, where M_e is the mass of the extremal configuration. We determine the Hawking temperature and a modified Area Law taking into account the extended nature of the source.

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I. INTRODUCTION

Relativistic, self-gravitating matter shells have been thoroughly investigated in different sectors of theoretical physics: “... from cosmic inflation to hadronic bags” [1]. Remarkable applications of gravitational shell models can be found in the framework of inflationary cosmology, where, both the “birth” and the evolution of vacuum bubbles can be effectively described in terms of the dynamics of the boundary surface engulfing a false vacuum domain [1–4]. Matter shells in general relativity are modeled as “zero-thickness” membranes endowed with some characteristic tension determined by the underlying classical, or quantum, physics. Neglecting the real width of the mass-energy distribution affects Einstein equations by introducing a surface of discontinuity in the background spacetime. This approximation allows to encode all the dynamics in the matching condition between the inner and outer geometries [5]. Furthermore, contracting matter shells provide useful analytic toy-models of collapsing massive bodies leading to black hole formation. In the spherically symmetric case the only dynamical degree of freedom is given by the shell radius and the system can be quantized according with the standard principle of quantum mechanics. In this framework, self-gravitating quantum shells open a window over the still “murky” quantum features of evaporating mini black holes [6–10].

In this paper we are going to investigate the static, final stage, of collapsed matter shell in the presence of a fundamental *minimal length* forbidding the shell to contract into a singular matter point. The emergence of a minimal length, as a new fundamental constant of Nature on the same ground as c and \hbar , is a general feature of different approaches to quantum gravity

[11–13].

In a series of papers we showed as the very concept of “point-particle” is meaningless if there exists a lower bound to physically measurable lengths [14–18]. This basic notion can be encoded into the Einstein equations through a proper choice of the energy momentum tensor [19–29]. The most remarkable outcomes of this procedure are the disappearance of curvature singularities in the solutions of the Einstein equations [30], a regular behavior of the Hawking temperature which allows to determine the physical character of the evaporation remnant [31–37], and a different form of the relation between entropy and area of the event horizon reproducing the celebrated Area Law for large, semi-classical, black holes.

The paper is organized as follows. In section II we will derive new solutions of Einstein equations describing, spherically symmetric, static, self-gravitating, thick matter layers, in the presence of a fundamental minimal length. These type of objects are modeled by means of a proper modification of the source for “non-commutative geometry inspired”, regular black holes. We replace the Gaussian profile of mass-energy, peaked around the origin, with higher moments of Gaussian distribution with maxima shifted at finite distance from the origin. The finite width of the distribution is determined by the minimal length. In the case of lump-type objects, the minimal length measures the spread of mass-energy around the origin and removes the curvature singularity. For finite width matter layers, with energy density vanishing at short distance, not only the central curvature singularity is removed, but the extrinsic curvature discontinuity between “inner” and “outer” geometry is cured, as well. Spacetime geometry is continuous and differentiable everywhere and smoothly interpolates between Minkowski metric near the center, and Schwarzschild spacetime far away from the matter layer.

As in the cases previously discussed, black hole type solutions exist only for “heavy” shells, i.e. $M \geq M_e$, where M_e is the mass of the extremal configuration. “Light” matter layers with $M < M_e$ will settle down in a smooth solitonic type configuration with no horizons or curvature singularity.

In section III we study the thermodynamic properties of black hole solutions and determine both the Hawking Temperature, T_H , and the relation between entropy and area of the horizon. We recover the celebrated Area Law in the limit of large, semi-classical, black holes. On a general ground, we find the leading term is one fourth of the area but in units of an “effective” gravitational coupling constant, $G_N(r_+)$, depending from the radius of black hole. For “large” r_+ , the Newton Constant is recovered. Finally, in section IV we will draw the conclusions.

II. THICK SHELLS

In a previous series of papers we solved the Einstein equations including the effects of a minimal length in a proper energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}, \quad (1)$$

where $T_{\mu\nu} = \text{diag}(\rho, p_r, p_\perp, p_\perp)$. The energy density was chosen to be a minimal width Gaussian distribution

$$\rho = \rho_0(r) \equiv \frac{M}{(4\pi\theta)^{3/2}} e^{-r^2/4\theta} \quad (2)$$

representing a “blob-like” object, centered around the origin, with a characteristic extension given by $l \propto \sqrt{\theta}$. The radial pressure p_r is fixed by the equation of state $p_r = -\rho$ reproducing the deSitter “vacuum” equation of state at short distance. This is a key feature to build up a regular, stable configuration, where the negative pressure balances the gravitational pull. In other words, the singularity theorem is evaded by a violation of null energy condition triggered by the short distance vacuum fluctuations.

Finally, the tangential pressure p_\perp is obtained in terms of ρ by the divergence free condition $\nabla_\mu T^{\mu\nu} = 0$.

As the source is static and spherically symmetric, the line element can be cast in the form

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 \Omega^2, \quad (3)$$

with

$$f(r) = 1 - \frac{2G_N m(r)}{r}. \quad (4)$$

The cumulative mass distribution $m(r)$ is given by

$$m(r) = 4\pi \int_0^r dr' (r')^2 \rho_0(r'). \quad (5)$$

We notice that the parameter M corresponds to the total mass energy of the system, namely

$$M = \lim_{r \rightarrow \infty} m(r). \quad (6)$$

The above profile cures the usual Dirac delta (singular) distribution associated to a point-particle, and leads to a family of regular black hole solutions [19–28].

We are going now to adapt the above approach to an object which is *shell-like*, rather than point-like. For this reason, we recall that the Gaussian distribution admits moments of all orders. The regular black holes we found from Gaussian distribution are just gravitational objects corresponding to the zero-th moment of the distribution. Here we want to explore the nature of the solutions emerging from higher moments of the Gaussian, namely normalized matter distributions of the kind

$$\rho_k(r) \equiv M \frac{r^{2k} e^{-r^2/4\theta}}{4^{k+2} \pi \theta^{k+3/2} \Gamma(k + 3/2)} \quad (7)$$

where $k = 0, 1, 2, \dots$ is a natural number. We are considering the case of even moments only. This choice is motivated by the request of having a rapidly vanishing energy profile at the origin, corresponding to the case of “empty” shells. On the contrary odd moments may admit a linearly vanishing energy profile, which gives rise to a gravitational object which does not properly describe a matter shell. We stress that the choice of even moments does not modify any of the following conclusions for both the geometric and the thermodynamic properties of the solutions.

For $k = 0$ the function ρ_k turns into the Gaussian distribution, centered around the origin, while for $k \geq 1$ the matter distribution is more and more diluted near the origin, being peaked at $r_M = 2\sqrt{k\theta}$. As a result the density function (7) describes a whole family of “mass-degenerate” shells, with the same M , but concentrated at a distance given by r_M (see Fig. 1). Eq. (7) discloses further properties of the minimal length, which assumes a

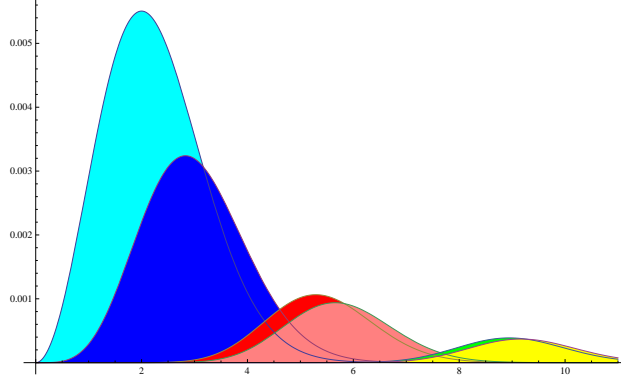


FIG. 1: Plot of the radial density profile $\rho(r)$ as a function of r in $\sqrt{\theta}$ -units for $k = 1$ (cyan), $k = 2$ (blue), $k = 7$ (red), $k = 8$ (pink), $k = 20$ (green), $k = 21$ (yellow). The function $\rho(r)$ has been normalized so that its integration over a spherical volume gives 1.

new intriguing meaning. In the case of point-particle, $\sqrt{\theta}$ represents the spread of the object around the origin. For matter shells, we see $\sqrt{\theta}$ relates to the shell thickness or, in other words, a measure of the intrinsic fuzziness of the layer. Just as a particle cannot be exactly localized at a single point, we cannot have zero-width layers, as well. Thus, as the matter distribution is smooth everywhere, there is discontinuity in the extrinsic curvature between the “inner” and outer “geometry”. No matching condition is required and we can look for a single, smooth, metric inside, across and outside the matter layer.

We can define the distance between two shells corresponding to different moments as the distance between the peaks:

$$\frac{\Delta r_M}{2\sqrt{\theta}} = \sqrt{k+1} - \sqrt{k}. \quad (8)$$

We see that for higher moments Δr_M vanishes as we are at length scale much larger than $\sqrt{\theta}$ and the relative distance cannot be resolved anymore.

A stable solution of the Einstein equations can be obtained by sourcing the gravitational field by the energy momentum tensor of an anisotropic fluid: the choice $p_r = -\rho_k$ for the matter equation of state allows to have $g_{00} = -g_{rr}^{-1}$. Furthermore, the hydrodynamic equilibrium equation will give p_t in terms of ρ_k , while the metric itself results to be independent from p_t . The solution of the Einstein equations reads in geometric units, $c = 1$, $G_N = 1$:

$$ds^2 = - \left(1 - \frac{2m(r)}{r} \right) dt^2 + \left(1 - \frac{2m(r)}{r} \right)^{-1} dr^2 + r^2 d\Omega^2 \quad (9)$$

$$m(r) \equiv 4\pi \int_0^r dr' r'^2 \rho(r') = M \frac{\gamma(k + 3/2; r^2/4\theta)}{\Gamma(k + 3/2)} \quad (10)$$

where $\gamma(k + 3/2; r^2/4\theta)$ is the lower, incomplete, Euler Gamma Function. By taking into account the asymptotic form of $\gamma(k + 3/2; r^2/4\theta)$ for small argument, one finds that the metric (9) is essentially flat near the origin, the contrary to what happens for the case $k = 0$

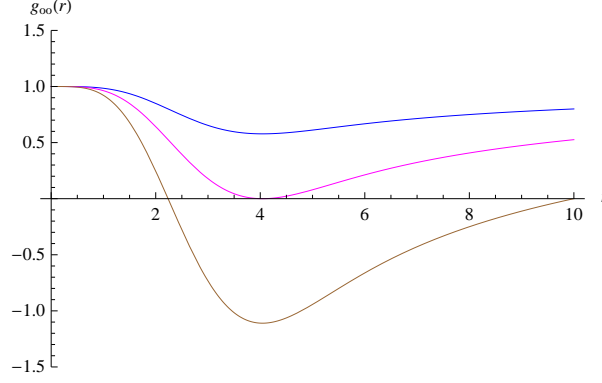


FIG. 2: Plot of $g_{00}(r)$ as a function of r in $\sqrt{\theta}$ -units for fixed value of $k = 1$ and different values of the mass M , i.e. $M = 1$ (blue), $M = M_e$ (magenta) and $M = 5$ (brown). Here $M_e \simeq 2.36976$ is the extremal mass for the case $k = 1$.

in which a regular deSitter core forms. Indeed one finds

$$f(r) \simeq 1 - \frac{1}{2k+3} \left[\frac{4M}{r \Gamma(k+3/2)} \right] \left(\frac{r^2}{4\theta} \right)^{k+3/2}, \quad (11)$$

$$ds^2 = - \left(1 - O(r^{2k+2}) \right) dt^2 + \left(1 - O(r^{2k+2}) \right)^{-1} dr^2 + r^2 d\Omega^2. \quad (12)$$

The asymptotic behavior (12) can be seen as a generalization of the “Gauss Theorem” : inside an empty, classical, thin shell of matter the Newtonian gravitational field is zero, i.e. spacetime is flat. Introducing a finite width density, smoothly decreasing both toward the origin and space-like infinity, causes small deviations from perfect flatness.

Going back to (9), the presence of event horizons is read from the zeroes of g_{rr}^{-1} (see Fig. 2). Due to the non-trivial structure of the metric, it is more convenient to look at the “horizon equation” in the form

$$M \equiv U(r_H), \quad U(r_H) \equiv \frac{r_H}{2} \frac{\Gamma(k+3/2)}{\gamma(k+3/2; r_H^2/4\theta)}. \quad (13)$$

The problem of finding the horizons in the metric (9) is mapped in the equivalent problem of determining the turning-points for the motion of a “test-particle” of energy M subject to the “potential” $U(r_H)$. The intersection(s) between a line $M = \text{const.}$ and the plot of the function $U(r_H)$, in the plane M - r_H , represent the allowed radii of inner/outer event horizons (see Fig. 3). The asymptotic behavior of $U(r_H)$ is obtained from the corresponding approximate forms of γ :

$$U(r_H) \approx \frac{r_H}{2}, \quad r_H \gg \sqrt{\theta} \quad (14)$$

$$U(r_H) \approx \sqrt{\theta} \Gamma(k+5/2) \left(\frac{r_H^2}{4\theta} \right)^{-(k+1)}, \quad r_H \ll \sqrt{\theta}. \quad (15)$$

Thus, $U(r_H)$ is a convex function with a single minimum determined by the condition

$$\frac{dU}{dr_H} = 0 \longrightarrow \frac{r_e^{2k+3}}{2^{2k+2}\theta^{k+3/2}} = \gamma(k+3/2; r_e^2/4\theta) e^{r_e^2/4\theta}. \quad (16)$$

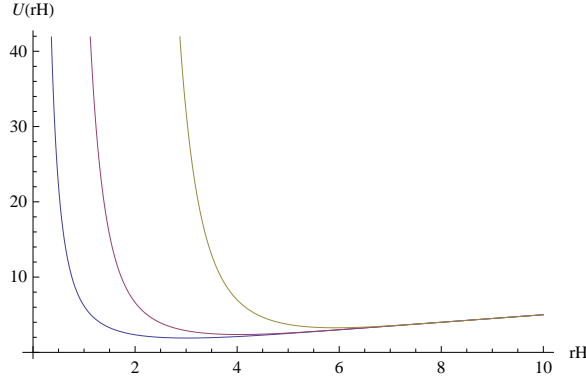


FIG. 3: Plot of $U(r_H)$ as a function of r_H in $\sqrt{\theta}$ -units for $k = 0$ (blue), $k = 1$ (magenta) and $k = 4$ (brown).

The radius r_e represents the size of an *extremal* configuration with a couple of degenerate horizons: $r_- = r_+ \equiv r_e$. Once r_e is numerically determined from (16), one gets the corresponding mass M_e from the potential:

$$M_e = U(r_e) = U_{min}. \quad (17)$$

An order of magnitude estimate for r_e and M_e can be obtained by keeping only the leading theta-term:

$$r_e \propto \sqrt{\theta} \longrightarrow M_e \propto \frac{\sqrt{\theta}}{L_{Pl.}} M_{Pl.} \quad (18)$$

where $L_{Pl.}$ is the Planck length, i.e., $L_{Pl.} = M_{Pl.}^{-1} = \sqrt{G_N}$. This estimate suggests that (near/)extremal configurations are close to a full quantum gravity regime. As such, they are appropriate candidates to describe the end-point of the Hawking evaporation process where the semi-classical description breaks down.

In summary:

- for $M > M_e$ we find a geometry with non-coincident inner and outer horizons $r_- < r_+$. It is worth to remark that there is no curvature singularity in $r = 0$. Spacetime is flat near the origin.
- For $M = M_e$ we have an *extremal* configuration with a pair of degenerate horizons, $r_- = r_+ \equiv r_e$.
- For $M < M_e$ there are neither horizons nor curvature singularities. The metric is smooth and regular everywhere. The shell is too light and diluted to produce any relevant alteration of the spacetime fabric. After collapse it will settle into a sort of solitonic object with no horizon of curvature singularity.

Massive layers will produce horizons, but no curvature singularities. This is a crucial point which marks a departure with respect to all the existing literature. Our approach must not be confused with previous contributions in which generalized matter shells containing polytropic and Chaplygin gas cannot ultimately resolve the emergence of singularities [38].

We can estimate the size of these objects by solving iteratively the equation (13) and truncating the procedure at the first order in the expansion parameter $\exp(-M^2/\theta)$:

$$r_+ \simeq 2M \left[1 - \frac{(M^2/\theta)^{k+1/2}}{\Gamma(k+3/2)} e^{-M^2/\theta} \right]. \quad (19)$$

This quantity can be compared with the shell *mean radius*

$$\langle r \rangle \equiv \frac{4\pi}{M} \int_0^\infty dr r^2 \rho(r) = 2\sqrt{\theta} \frac{\Gamma(k+2)}{\Gamma(k+3/2)}. \quad (20)$$

We notice that for $k \gg 1$ the leading contribution to the ratio $r_+/\langle r \rangle$ is given by

$$\frac{r_+}{\langle r \rangle} \approx \frac{M}{\sqrt{\theta}} \frac{1}{\sqrt{k+1}}. \quad (21)$$

Thus, for an assigned M the ratio decreases with k . The horizon radius is determined by the total mass energy M and is weakly k -dependent. On the contrary, $\langle r \rangle \propto \sqrt{k+1} \sqrt{\theta}$ and grows with k , which means that for higher moments the horizon is surrounded by a cloud of matter.

III. THERMODYNAMICS

Massive shells will collapse into black holes described by the line element (3),(4). This is not the end of story as these objects are semi-classically unstable under Hawking emission. We are now ready to study the thermodynamic properties of these solutions starting from the Hawking temperature:

$$T_H = \frac{1}{4\pi r_+} \left[1 - \frac{r_+^{2k+3}}{2^{2k+2} \theta^{k+3/2}} \frac{e^{-r_+^2/4\theta}}{\gamma(k+3/2; r_+^2/4\theta)} \right]. \quad (22)$$

It can be easily verified that T_H is vanishing for the extremal configuration $T_H(r_+ = r_e) = 0$. For $k = 0$ we obtain the temperature of the black hole as in [21]. The behavior of the temperature can be found in Fig. 4. We notice that the regularity of the manifold for any k leads to a cooling down of the horizon in the terminal phase of the Hawking process. At $r_+ = r_{max}$ the presence of a maximum temperature corresponds to an infinite discontinuity the heat capacity which is usually interpreted as the signal of a “change of state” for the system. For $r > r_{max}$ the black hole is thermodynamically unstable and increases its temperature by radiating away its own mass. After crossing r_{max} , i.e., for $r_+ < r_{max}$ the black hole enters a stability phase asymptotically ending into a degenerate (zero-temperature) extremal configuration. By increasing k we just lower down the maximum temperature. As a result the solution is unaffected by any relevant quantum back reaction as already proved for the case $k = 0$ in [21].

A further important consequence of $T_H \leq T_{max}$ is that (in extra-dimensional models) the black hole mainly radiates on the brane [43], it never becomes hot enough to warm-up the bulk in a significant way.

The area-entropy law can be recovered from the relation

$$dM = T_H dS \quad (23)$$

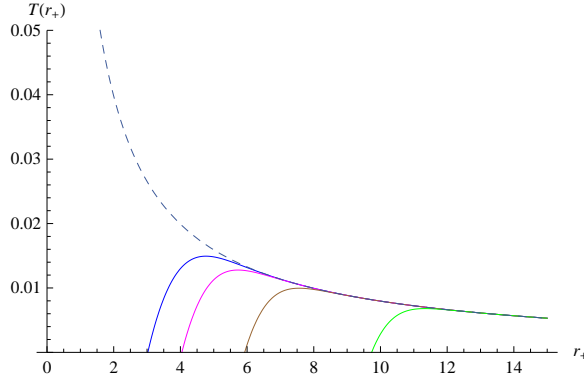


FIG. 4: Plot of $T(r_+)$ as a function of r_+ in $\sqrt{\theta}$ -units for $k = 0$ (blue), $k = 1$ (magenta) and $k = 4$ (brown), $k = 15$ (green). The dashed line represent the classical case, i.e. $\theta = 0$.

TABLE I: Some values of parameters of the matter shells

| | r_e | M_e | T_{max} | r_{max} |
|----------|--------|--------|-----------|-----------|
| $k = 0$ | 3.0224 | 1.9041 | 0.014937 | 4.76421 |
| $k = 1$ | 4.0431 | 2.3698 | 0.012783 | 5.72632 |
| $k = 4$ | 5.9269 | 3.2647 | 0.009960 | 7.56069 |
| $k = 15$ | 9.7347 | 5.1245 | 0.006799 | 11.3419 |

where dS is the horizon entropy variation triggered by a variation dM in the total mass energy M . In order to translate the first law of black hole thermodynamics (23) into a relation involving the area of the event horizon, we need to write mass energy variation as

$$dM = \frac{\partial U}{\partial r_+} dr_+ \quad (24)$$

and to take into account that the minimum of $U(r_+)$ is the mass of the extremal black hole. Thus, when integrating (24) the lower integration limit is the radius of the extremal configuration

$$S = \int_{r_+}^{r_e} dx \frac{1}{T_H} \frac{\partial U}{\partial x} = 2\pi \Gamma(k + 3/2) \int_{r_+}^{r_e} dr \frac{r}{\gamma(k + 3/2; r^2/4\theta)}, \quad (25)$$

where we have inserted (22) and (24) into (25). By performing the integral one gets

$$S = \pi \Gamma(k + 3/2) \left(\frac{r_+^2}{\gamma(k + 3/2; r_+^2/4\theta)} - \frac{r_e^2}{\gamma(k + 3/2; r_e^2/4\theta)} \right) + \pi \Gamma(k + 3/2) \int_{r_+}^{r_e} dr r^2 \frac{\gamma'}{\gamma^2}. \quad (26)$$

The first term can be written in terms of the area of the event horizon as

$$S = \frac{1}{4G_N} \Gamma(k + 3/2) \left(\frac{A_H}{\gamma(k + 3/2; r_+^2/4\theta)} - \frac{A_e}{\gamma(k + 3/2; r_e^2/4\theta)} \right) + \dots \quad (27)$$

and represents the “Area Law” in our case. We have re-inserted the Newton constant into (27) for reasons to become clear in a while. The standard form, which is one fourth of the area, is recovered in the large black hole limit, $r_H \gg \sqrt{\theta}$.

Once (27) is written in natural units, we can define an *effective* Newton constant as

$$G_N \rightarrow G_N(r_+) \equiv G_N \frac{\gamma(k + 3/2; r_+^2/4\theta)}{\Gamma(k + 3/2)} \quad (28)$$

and introduce a “modified” Area Law as

$$S(r_H) = \frac{\pi r_H^2}{4G_N(r_H)}. \quad (29)$$

The interesting feature of $G_N(r_H)$ is to be “*asymptotically free*” in the sense that it is smaller than G_N , which represents the asymptotic value of the gravitational coupling for large black holes only [48].

Finally, the second term in (26), gives exponentially small corrections to the leading area term. It can be analytically computed for $k \gg 1$, but it does not introduce any relevant new effect.

IV. CONCLUSIONS

In this paper we derived new static, spherically symmetric regular solutions of Einstein equations, describing the final state of collapsing matter shells in the presence of an effective minimal length. This derivation is the result of a long path started with the quest of quantum gravity regularized black hole solutions [30]. We showed that our previous discoveries of noncommutative geometry inspired solutions correspond to the simplest case within a larger class of regular gravitational objects. This class, the family of gravitational shells here presented, has peculiar properties which descend from the common key feature of the absence of curvature singularities. Such properties are: the possibility of horizon extremisation even for the neutral, non-rotating case, the existence of a minimum amount of mass energy below which horizons do not form, the occurrence of a phase transition from a thermodynamically unstable classical phase to a thermodynamically stable cooling down in the final stage of the horizon evaporation. As for the case of regular black holes, these matter shells correspond to final configurations of dynamical processes of gravitational collapse in which matter cannot be compressed below a fundamental length scale. The above properties are also common to other models of quantum geometry, that have been derived by means of different formalisms. Specifically the shell profile of the energy density resembles what one has in the case of loop quantum black holes, with consequent thermodynamic similarities [44–47].

The study of these objects is far from being complete. We believe that these new solutions can have repercussions in a variety of fields. Here we just mention that these regular shells could affect the stability of the deSitter space, at least during inflationary epochs [27] as well as they could lead to new insights in the physics of nuclear matter via the gauge/gravity duality paradigm [37].

Acknowledgments

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